Mathematics I (H) Dated :- 16/04/21
Part I.

Countable and Uncountable Set.

Definitions :-

Denumerable set :- A set A is called denumerable set if $A \sim N$.
i.e. $\{x_1, x_2, x_3, ---, x_n, ---\}$ is denumerable because this set $\sim N$ under the map $a_n \to n$.
In other words a set A is called denumerable if its elements can be put to one-one correspondence with the set of natural numbers.

Countable sets :- A set A is called countable if it is finite or denumerable
i.e. $A = \{1, 2, 3, 4, 5, 6, 7\}$. it is finite so by definition it is countable.
$A = \{x_1, x_2, x_3, --- x_n ---\}$ so that A is denumerable so by definition A is countable.

Uncountable set :- A set A is called an uncountable set if A is an infinite set or A is not countable.
i.e. set of real number is uncountable.

Finite set :- A set A is called a finite set if it is equivalent to $\{1, 2, 3, --- n\}$ for some value of $n \in N$.
i.e. $\{1, 2, 3, 4, 5, 6, 7\}$, $\{a_1, a_2, a_3, a_4, --- a_{10}\}$ are finite set.
Infinite set :- A set A is called an infinite set if it is a equivalent to the proper subset of

Theorem:– Show that set of rational number is enumerable.

Proof:– We know that rational number is positive as well as negative.

Here we first show that the set of all positive rational number is denumerable.

For this we list the positive rational numbers is an infinite number of rows with the nth row containing all positive rationals with n as denominater but leave out all these which already occuse is the positive row.

$$\frac{1}{1},\ \frac{2}{1},\ \frac{3}{1},\ \frac{4}{1}$$

$$\frac{1}{2},\ \frac{3}{2},\ \frac{5}{2},\ \frac{7}{2}$$

$$\frac{1}{3},\ \frac{2}{3},\ \frac{4}{3},\ \frac{5}{3}$$

$$\frac{1}{4},\ \frac{3}{4},\ \frac{5}{4},\ \frac{7}{4}$$

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Now we arrange above as diagonally as follows

$$= \left\{1, \frac{1}{2}, \frac{2}{1}, \frac{1}{3}, \frac{3}{2}, \frac{3}{1}, \frac{1}{4}, \frac{2}{3}, \frac{5}{2}, \frac{4}{1} --- \right\}$$

In the same manner we arrange the set of negative rationals as follows

$$= \left\{-1, \frac{-1}{2}, \frac{-2}{1}, \frac{-1}{3}, \frac{-3}{2}, \frac{-3}{1}, \frac{-1}{4}, \frac{-2}{3}, \frac{-5}{2}, \frac{-4}{1} --\right.$$

In above two listed series, we set that all the rational numbers is listed in the form of the infinite sequence

$$A_n = \left\{0, 1, -1, \frac{1}{2}, \frac{-1}{2}, \frac{2}{1}, \frac{-2}{1}, \frac{1}{3}, \frac{-1}{3} --- \right\}.$$

Here the set of all rationals whose denominator belongs